Communication Link Budget Analysis

Reference: Communication Systems, 4/e, S. Haykin

Background Equations

Average probability of symbol error for coherent *M*-ary PSK

$$P_e = \operatorname{erfc}\left(\sqrt{\frac{E}{N_0}}\sin\left(\frac{\pi}{M}\right)\right) \tag{6.47}$$

where it is assumed that $M \geq 4$. E is energy per symbol, N_0 is the noise spectral density, and $\operatorname{erfc}(x)$ is complementary error function of x.

Noise spectral density may be expressed as

$$N_0 = kT_e \tag{1.94}$$

where k is Boltzmann's constant and T_e is the equivalent noise temperature of the receiver. *The equivalent noise temperature is defined as the temperature at which a noisy resistor has to be maintained such that, by connecting the resistor to the input of a noiseless version of the system, it produces the same available noise power at the output of the system as that produced by all the sources of noise in the actual system.*

Link margin

$$M(\text{dB}) = \left(\frac{E_b}{N_0}\right)_{\text{rec}}(\text{dB}) - \left(\frac{E_b}{N_0}\right)_{\text{req}}(\text{dB}) \tag{8.2}$$

Clearly the larger we make the link margin M, the more reliable is the communication linkk. However, the increased reliablity of the link is attained at the cost of a higher E_b/N_0.

Friis free-space equation

$$P_r = P_t G_t G_r \left(\frac{\lambda}{4\pi d}\right)^2 \tag{8.14}$$

where G_r is the power gain of the receiving antenna, G_t is the power gain of the transmitting antenna, P_t is the transmitted power, λ is the wavelength, and d is the distance between the transmitting and receiving antennas.

The overall equivalent noise temperature of the cascade connection of any number of noisy two-port networks

$$T_e = T_1 + \frac{T_2}{G_1} + \frac{T_3}{G_1 G_2} + \frac{T_4}{G_1 G_2 G_3} + \cdots \tag{8.30}$$

where T_1, T_2, T_3, … are the equivalent noise temperatures of the individual networks, and G_1, G_2, G_3, … are the available power gains, respectively. Equation (8.30) is known as the Friis formula.

Example 8.1 Noise Temperature of Earth-Terminal Receiver

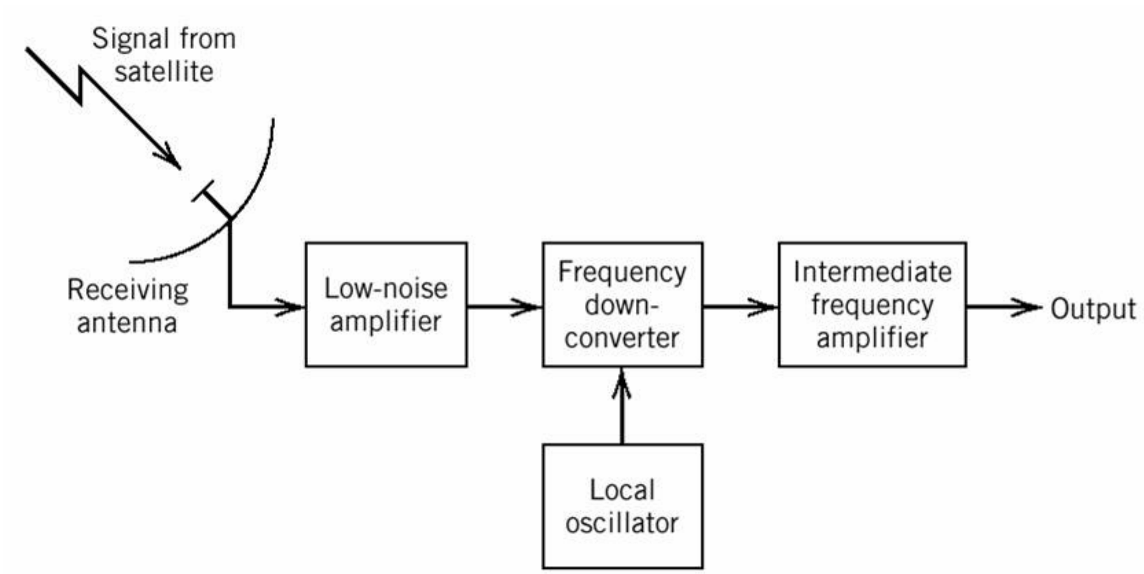

Figure 8.10 Block diagram of earth terminal receiver

Figure 8.10 shows a typical earth-terminal receiver, consisting of a low-noise radio-frequency (RF) amplifier (LNA), frequency down-converter (mixer), and intermediate frequency (IF) amplifier. The equivalent noise temperatures of these components, including the receiving antenna, are

$$T_{\text{antenna}} = 50\,\text{K}$$
$$T_{\text{RF}} = 50\,\text{K}$$
$$T_{\text{mixer}} = 500\,\text{K}$$
$$T_{\text{IF}} = 1000\,\text{K}$$

The available power gains of the two amplifiers are

$$G_{\text{RF}} = 200 = 23\,\text{dB}$$
$$G_{\text{IF}} = 1000 = 30\,\text{dB}$$

To calculate the equivalent noise temperature of the receiver, we use Equation (8.30), obtaining

$$
\begin{aligned}
T_{e=} &= T_{\text{antenna}} + T_{\text{RF}} + \frac{T_{\text{mixer}} + T_{\text{IF}}}{G_{\text{RF}}} \\
&= 50 + 50 + \frac{500 + 1000}{200} \\
&= 107.5\ \text{K}
\end{aligned}
$$

Example 8.2 Downlink Budget Analysis of a Digital Satellite Communication System

In a digital satellite communication system, one of the key elements in the overall design and analysis of the system is the downlink power budget, which is usually more critical than the uplink power budget because of the practical constraints imposed on the downlink power and satellite antenna size. The example presented here addresses a sample downlink budget analysis, assuming that any required uplink power (within limits) is available for satisfactory operation of the system.

The critical parameter to be calculated is the ratio of received carrier power-to-noise spectral density, denoted by $C/N_\circ$. According to the Friis free space equation (8.14), the average power received at the earth terminal to the average power P_t transmitted by the satellite is

$$
P_r = P_t G_t G_r \left(\frac{\lambda}{4\pi\, d} \right)^2
$$

where in this example, G_t is the power gain of the satellite antenna, G_r is the power gain of the receiving earth-terminal antenna, λ is the carrier wavelength for the downlink, and d is the distance between the satellite and the earth terminal. Given that the equivalent noise temperature of the system is T_e, we may use equation (1.94) of Chapter 1 to express the noise spectral density N_0 as kT_e, where k is Boltzmann's constant. Moreover, from Equation (8.10) we note that $P_t G_t$ is equal to the EIRP of the satellite. Hence, dividing P_r by N_0, we may express the C/N_0 ratio for the downlink as

$$
\left(\frac{C}{N_0} \right)_{\text{downlink}} = (\text{EIRP})_{\text{satellite}} \left(\frac{G_r}{T_e} \right)_{\text{earth terminal}} \left(\frac{\lambda}{4\pi\, d} \right)^2 \frac{1}{k} \qquad (8.31)
$$

For a given satellite system, the free-space loss $(4\pi\, d/\lambda)^2$ is a constant. Viewing the system from the earth terminal, we see from Equation (8.31) that the C/N_0 ratio is proportional to G_r/T_e may therefore be used to assess the quality of an earth terminal; it is usually shortened to the G/T ratio, which is referred to as the figure of merit of the receiving earth terminal. Thus, rewriting the formula (8.31) for the C/N_0 ratio measured in decibels,

1. $(\text{EIRP})_{\text{satellite}}$ measured in dBW, where dBW denotes decibels referenced to 1 watt, that is, 0 dBW.
2. $(G/T)_{\text{earth terminal}}$, measured in dB/K, where K refers to degree Kelvin.
3. $L_{\text{free space}}$, denoting the free-space loss $10\log_{10}(4\pi d/\lambda)^2$ in dB.
4. $-10\log_{10} k$, representing the gain in dBW/K-Hz due to division by the Boltzmann's constant $k = 1.38\times10^{-23}\ \text{joules/K}$.

Table 8.1 Downlink power budget for Example 8.2

Variable	Value
EIRP	+46.5 dBW
G/T ratio	+24.7 dB/K
Free-space loss	-206 dB
Boltzmann constant	+228.6 dBW/K-Hz
C/N_0	93.8 dB-Hz

Table 8.1 presents the values of those four terms for the downlink of a typical domestic digital satellite communication system, based on the following:

1. The transponder is operated at its maximum output power (i.e., no power backoff is employed), yielding an EIRP of 46.5 dBW.

2. The receiving earth terminal uses a 2m-dish antenna with a power gain $G = 45\ \text{dB}$, and the receiver is configured as in Example 8.1 with equivalent temperature $T = 107.5\ \text{K}$. Hence,

$$\begin{aligned}\frac{G}{T} &= 45 - 10\log_{10} 107.5\\ &= 45 - 20.3\\ &= 24.7\ \text{dB/K}\end{aligned}$$

3. The free space loss is

$$L_{\text{free space}} = 92.4 + 20\log_{10} f + 20\log_{10} d\ \ \text{dB} \tag{8.32}$$

where the downlink carrier frequency f is in GHz and the distance d between the satellite and the earth terminal in kilometers. For a geostationary satellite, the distance between the satellite and an earth terminal lies in the range 36000 to 41000 km. Thus choosing $d = 40000\ \text{km}$ and assuming $f = 12\ \text{GHz}$, the use of Equation (8.32) yields

$$\begin{aligned}L_{\text{free-space}} &= 92.4 + 20\log_{10} 12 + 20\log_{10} 40000\\ &= 92.4 + 21.6 + 92.0\\ &= 206\ \text{dB}\end{aligned}$$

4. With the Boltzmann's constant $k = 1.39 \times 10^{-23}$ joule/K, its contribution to the C/N_0 ratio is

$$-10\log_{10} k = 10\log_{10} 1.38 \times 10^{-23}$$
$$= 228.6 \text{ dBW/K - Hz}$$

Totaling the gains and losses, we thus get

$$\left(\frac{C}{N_0}\right)_{\text{downlink}} = 93.8 \text{ dB - Hz}$$

The received downlink value of the C/N_0 ratio may also be expressed in terms of the required value of the bit energy-to-noise spectral density ratio, $\left(E_b/N_0\right)_{\text{req}}$ dB at the receiving earth terminal as (see Equation (8.2))

$$\left(\frac{C}{N_0}\right)_{\text{downlink}} = \left(\frac{E_b}{N_0}\right)_{\text{req}} + 10\log_{10} M + 10\log_{10} R \text{ dB} \tag{8.33}$$

where $10\log_{10} M$ is the link margin in decibels, and R is the data rate in b/s. The link margin allows for excess rain losses in propagation and other power degradations. Typically, the link margin is selected as 4 dB for C-band, 6-dB for Ku-band, and higher for the higher K-band frequencies because of the higher rain losses. For operation at the Ku-band frequency of 12 GHz, we have chosen a link margin of 6 dB. Thus, using the value of $C/N_0 = 93.8$ dB - Hz calculated from the link budget, the link margin $10\log_{10} M = 6$ dB, and assuming $\left(E_b/N_0\right)_{\text{req}} = 12.5$ dB, the use of Equation (8.33) yields

$$10\log_{10} R = 93.8 - 12.5 - 6$$
$$= 75.3$$

Hence

$$R = 33.9 \text{ Mb/s}$$

Assuming the use of coherent 8-PSK for the transmission of digital data via the satellite, and substituting $\left(E_b/N_0\right) = 12.5$ dB in Equation (6.47) of Chapter 6, we find that the probability of symbol error $P_e = 0.6 \times 10^{-3}$.

To summarize, the digital satellite communication system analyzed in this example permits, under the worst operating conditions, data transmission on the downlink at a rate $R = 33.9$ Mb/s and with a probability of symbol error $P_e = 0.6 \times 10^{-3}$, assuming the use of 8-phase PSK.

Problems

8.10 In this problem we address the uplink power budget of the digital satellite communication system considered in Example 8.2. The parameters of the link are as follows:

Carrier frequency = 14 GHz
Power density at the TWT amplifier in saturation = $-81\,\mathrm{dBW/m^2}$
Satellite figure of merit, $G/T = 1.9\,\mathrm{dB/K}$
Distance of the satellite from the transmitting earth terminal $= 40000\,\mathrm{km}$

(a) Assuming no power backoff of the TWT, calculate the C/N_0 ratio at the satellite.
(b) Given that the data rate in the uplink is the same as that calculated for the downlink in Example 8.2, calculate the probability of symbol error incurred in the uplink allowing for a link margin of 6 dB. Compare your result with that in Example 8.2

8.11 The downlink C/N_0 ratio in a direct broadcast satellite (DBS) system is estimated to be 85 dB-Hz. The specifications of the link are:

Satellite EIRP = 57 dBW
Downlink carrier frequency = 12.5 GHz
Data rate = 10 Mb/s
Required E_b/N_0 at the receiving earth terminal = 10 dB
Distance of the satellite from the receiving earth terminal = 41000 km

Calculate the minimum diameter of the dish antenna needed to provide a satisfactory TV reception, assuming that the dish has an efficiency of 55 percent and it is located alongside the home where the temperature is 310K. For this calculation, assume that the operation of the DBS is essentially downlink-limited.